Baker and Kawashima Reply: Our letter [1] described numerical work and recognized for the first time that it should bracket the true value, g^*, of the renormalized coupling constant. As the lower limit is greater than zero, we have presented for the first time, what we feel to be a convincing demonstration that the renormalized coupling-constant is non-zero and therefore the long standing question of the validity of hyperscaling in the 3-dimensional Ising model is resolved beyond a reasonable doubt. The lower limit was argued to be the direct Monte Carlo estimate of $\lim_{L\to\infty} g(K_c, L) = G^*$, where L is the system size, K_c is the critical point value of the inverse temperature K, and $g(K, L)$ is the estimator function which leads to the renormalized coupling constant through the limits $\lim_{K\to K_c^-} \lim_{L\to\infty} g(K, L) = g^*$. The nature of the behavior of these two limits was explored by exact calculation for the two-dimensional Ising model previously [2] and $g^* \neq G^*$ was to be expected. In brief summary, $g* > G^*$, and for fixed L the decline from a value of $g(K, L) \approx g^*$ to $g(K_c, L) \approx G^*$ becomes progressively more precipitous as L increases. This behavior for large L is strongly supported in 3 dimensions by the Monte Carlo histogram method results of Tamayo and Gupta [3] who observe a very rapid decline in values of $g(K, L)$ as K_c is approached. Since the submission of our work [1] we have done a calculation of G^* [4] and found $G^* = 5.0 \pm 0.2$. Also Tamayo and Gupta [3] have refined their much more extensive analysis and they found $G^* = 5.23 \pm 0.01$, more than 500 standard deviation off zero!

The work [5] referred to in the comment of Kim [6] was a good step forward at the time, however it failed to recognize the possibility of a lower bound as described above. Therefore, in common with other work at that time, it could not answer in a convincing manner the counter claim that $g(K, \infty)$ vanished like $(K_c - K)^{\omega^*}$ where ω^* is of the order of a few hundredths. This effect is apparent only very close to the critical point. The issue of the decline of $g(K, L)$ over the range we studied is not in our view relevant to the critical point behavior. Since correlation length vanishes at infinite temperature, we plotted $(K/K_c)^{3/2} g(K, L)$ to account for this effect. This quantity is constant over our whole range within the accuracy of Kim and Patrascioiu's previous work [5], i.e., 3. In other words, they might have observed similar interesting variations if they had squeezed the error down to the same magnitude as we did in our letter, i.e., 0.3 at the largest. Therefore, his claim that $g(K)$ remains constant in the scaling regime in their previous work, from our view point, did not have a strong basis. Kim and Patrascioiu have computed their results for $L/\xi \approx 6$. Baker [2] has found in two-dimensions that $L/\xi = 7 \pm 1$ is necessary for one percent work. Kim's argument that the results of [5] agree with ours where they overlap within their rather large errors, may not be enough to exclude systematic errors of several percent. Reinforcing this concern is the work of Baker and Erpenbeck [7] in three dimensions who clearly showed the

$L/\xi = 3.8$ is too small for accurate work. We have advanced matters by using $L/\xi \approx 10$ which should reduce the systematic error to less than a percent and have computed results. The cost for achieving this goal should not be neglected. First, because of the cancellation of large terms in the estimation of the four-point function, the extra accuracy required to increase L/ξ from 6 to 10 is at least of a factor of $(10/6)^3 \sim 5$, which is equivalent to a factor of $5^2 = 25$ in terms of computational time. In addition, as we pointed out, we have improved the accuracy by an order of magnitude over that previously reported [5], which amounts to a factor of 100 in terms of computational time. Finally, considering also trivial increase in computation due to the increase in system size, i.e., another factor of 5, the effort to obtain our results is about 10,000 times as great as would have been required for results with $L/\xi \approx 6$ and with ten times larger statistical errors as in the work of Kim and Patrascioiu. This improvement was made possible due largely to our improved estimators.

George A. Baker, Jr.

Theoretical Division, University of California,
Los Alamos National Laboratory, Los Alamos, N.M.
87545

Naoki Kawashima

Department of Physics, University of Tokyo, Hongo,
Bunkyo, Tokyo, Japan

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[1] G. A. Baker, Jr. and N. Kawashima, Phys. Rev. Lett. **75**, 994 (1995).
[2] G. A. Baker, Jr., J. Stat. Phys. **77**, 955 (1994).
[3] P. Tamayo and R. Gupta, in preparation (1995). We are grateful for permission to cite this work in advance of publication.
[4] G. A. Baker, Jr. and N. Kawashima, accepted for publication in the Proceedings of "Hayashibara Forum '95, International Symposium on Coherent Approaches to Fluctuations" (World Scientific, Singapore, 1996).
[5] J.-K. Kim and A. Patrascioiu, Phys. Rev. D **47**, 2588 (1993).
[6] J.-K. Kim, previous comment.
[7] G. A. Baker, Jr. and J. J. Erpenbeck, *Springer Proceedings in Physics,* **78**, *Computer Simulation Studies in Condensed-Matter Physics VII*, D. P. Landau, K. K. Mon, and H.-B. Schuttler, eds., (Springer, Berlin, 1994), pp. 213-218.